Lesson 4-1: Polynomial Functions **L.O:** *I CAN determine roots of polynomial equations. I CAN apply the Fundamental Theorem of Algebra*.
Date: _______________

Polynomial in one variable – ***An expression of the form*** $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ **where the coefficients** $a_{0,} a_{1,\ldots\ldots} a_n$ **represent complex numbers,** a_n **is not zero, and n represents a nonnegative integer.**

Degree of a polynomial in one variable– The greatest exponent of the variable of the polynomial.

Leading Coefficient – The coefficient of the term with the highest degree.

Polynomial Function – A function y = P(x) where P(x) is a polynomial in one variable.

Zero– A value of x for which f(x)=0.

Polynomial Equation – A polynomial that is set equal to zero.

Root – A solution of the equation P(x)=0.

Imaginary Number – A complex number of the form a + bi where $b \neq 0$ $and\ \ i\, is\, the\, imaginary\, unit$. ***Note:*** $i^2 = -1, \sqrt{-1} = i$

Complex Number – Any number that can be written in the form a + bi, where a and b are real numbers and i ***is the imaginary unit.***

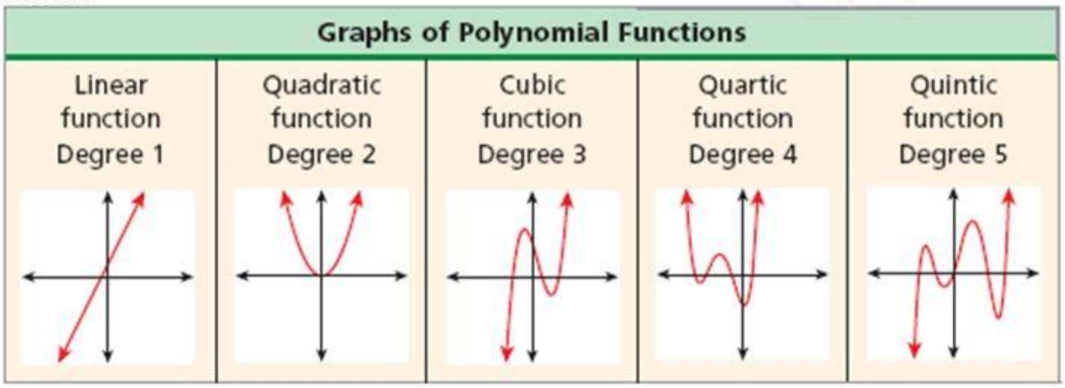
Pure imaginary Number – The complex number a + bi when a = 0 and $b \neq 0$.

Fundamental Theorem of Algebra – Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

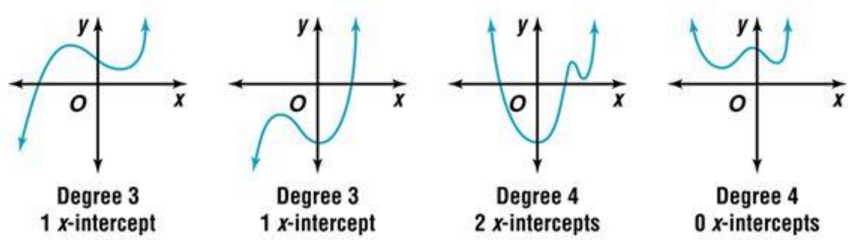
Corollary to the Fundamental Theorem of Algebra – Every polynomial P(x) of degree n (n>0) can be written as the product of a constant k $(k \neq 0)$ ***and n linear factors.*** $P(x) = k(x - r_1)(x - r_2)(x - r_3)\ldots.(x - r_n)$ ***thus, a polynomial equation of degree n has exactly n complex roots, namely*** $r_1, r_2, r_3, \ldots, r_n$.

******NOTE************The general shapes of the graphs of polynomial functions with positive leading coefficients and degree greater than 0 are shown below. These graphs also show the *maximum* number of times the graph of each type of polynomial may cross the x-axis.

Polynomial functions with positive lead coefficients and maximum number of real roots.



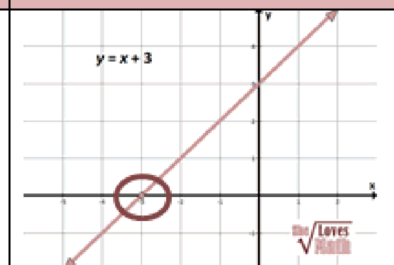
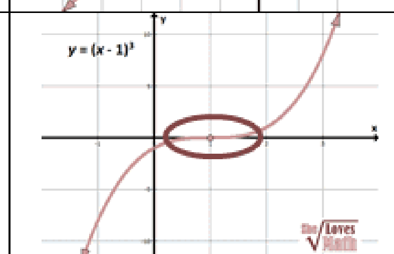
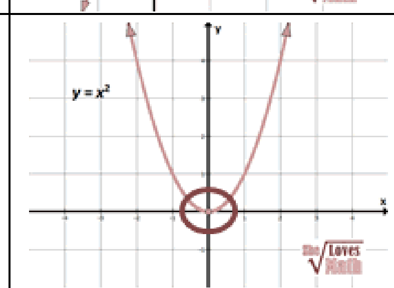
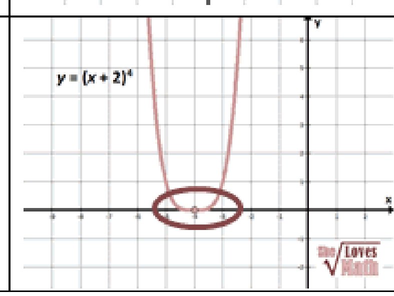
Third and fourth degree functions with real and imaginary roots.



******NOTE***** Since the x-axis only represents real numbers, imaginary roots cannot be determined by using a graph. The graphs below have the general shape of a third-degree function and a fourth-degree function. In these graphs, the third-degree function only crosses the x-axis once, and the fourth degree function crosses the x-axis twice or not all.

Graphs of Polynomial Functions: End Behavior & Turning Points

	Odd degree		Even degree	
Sign of Leading Coefficient	Positive	Negative	Positive	Negative
End behavior	↙ ↗	↖ ↘	↖ ↗	↙ ↘
Number of turning points	n - 1 at most; each turning point is a local (or global) max. or min.			

Factor(s)	Root	Multiplicity	Behavior	Example Graph
$(x+3)$	-3	**Odd:** 1	**Pass Through**	
$(x-1)^3$	1	**Odd:** 3, 5, … (the higher the odd degree, the flatter the "squiggle")	**"Squiggle" Pass Through**	
x^2	0	**Even:** 2	**Bounce** or **Touch**	
$(x+2)^4$	-2	**Even:** 4, 6, … (the higher the even degree, the flatter the bounce)	**"Flatter" Bounce**	

****NOTE****The graph of a polynomial function with odd degree must cross the x-axis at least once. The graph of a function with even degree may or may not cross the x-axis. If it does, it will cross an even number of times. Each x-intercept represents a real root of the corresponding polynomial equation.

****NOTE*** If you know the roots of a polynomial equation, you can use the corollary to the Fundamental Theorem of Algebra to find the polynomial equation. That is , if a and b are roots of the equation, the equation must be (x-a)(x-b) =0.

EX. 1 – END BEHAVIOR

1a. State the degree and leading coefficient of the polynomial. Then state the end behavior.

$f(x) = x^3 - 6x^2 + 10x - 8.$.

b. Determine whether 4 is a zero of f(x).

2a. State the degree and leading coefficient of the polynomial

$f(x) = 3x^4 - x^3 + x^2 + x - 1.$

b. Determine whether -2 is a zero of f(x).

3a. State the degree and the leading coefficient of the polynomial
$f(x) = 5m^2 + 8m^5 - 2$

b. Determine whether 5 is a zero of f(x).

Short Summary #1:

EX.2 – WRITING POLYNOMIAL EQUATIONS OF LEAST DEGREE WITH GIVEN ROOTS.

a. Write a polynomial equation of least degree with roots 2, 4i, and -4i.

- Does the equation have an odd or even degree? How many times does the graph of the related function cross the x-axis?

b. Write a polynomial equation of least degree with roots 2, 3i, and -3i.

- Does the equation have an odd or even degree? How many times does the graph of the related function cross the x-axis?

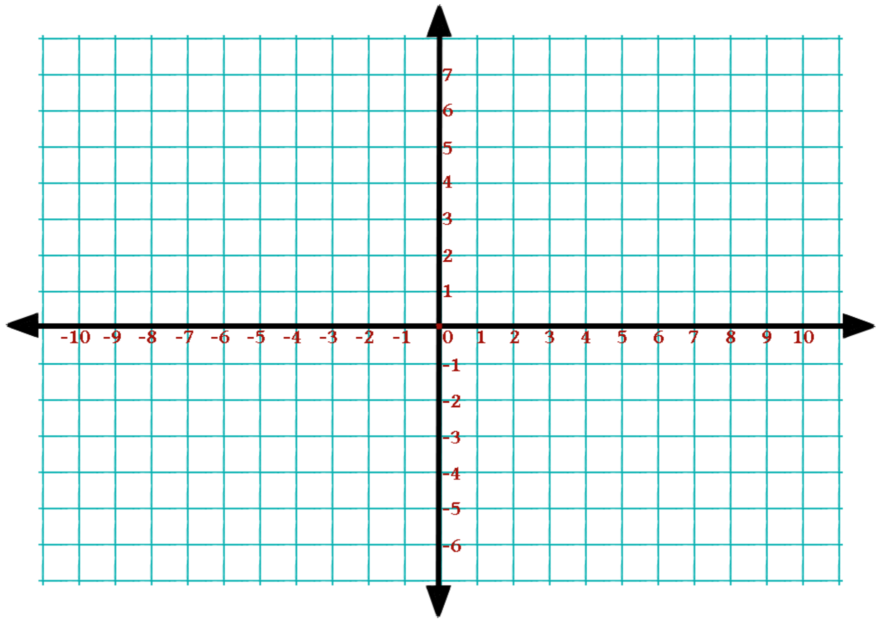
c. Write a polynomial equation of least degree with roots -3, 2i, and -2i.

b. Does the equation have an odd or even degree? How many times does the graph of the related function cross the x-axis?

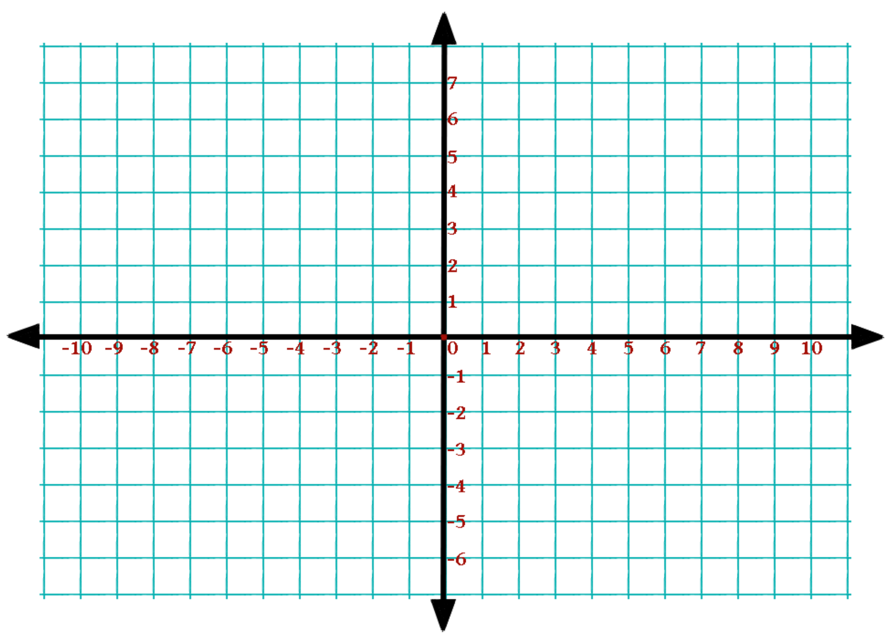
Short Summary #2:

EX.3 – FINDING THE ROOTS OF A POLYNOMIAL EQUATION

a.) State the number of complex roots of the equation $9x^4 - 35x^2 - 4 = 0$. Then find the roots and graph the related function.



b.) State the number of complex roots of the equation $32x^3 - 32x^2 + 4x - 4 = 0$. Then find the roots and graph the related function.



c. State the number of complex roots of the equation $t^3 + 2t^2 - 4t - 8 = 0$. Then find the roots and graph the related function.